B.Sc. Semester-III Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 32110 Course Code : SP/MTH/304/SEC-1

Course Title : Logic and Sets

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT–I

1. Answer any **five** of the following questions: 2×5=10

a) Find the truth table of $\neg p \wedge q$.

b) If A, B, C are subsets of S, then prove that $A-(B\cap C)=(A-B)\cup(A-C)$.

c) Prove that $A-B=B'-A'$, where A, B are subsets of S.

d) Translate the following sentence into symbolic forms:

There is a student in a school who can speak English but who does not know Hindi.

e) Determine the truth value of each of the following statements:

i) 4+2=5 and 6+3=9

ii) 4+5=9 and 1+2=4

f) Give an example of a relation which is transitive and symmetric but not reflexive.

g) If $aN=\{ax : x\in N\}$ then find $4N\cap 6N$ where N is the set of all natural numbers.

h) On the set {1, 2, 3, 4, 5} R is defined by aRb iff $a-b$ is divisible by 3. Find R and the range R^{-1}.

UNIT–II

2. Answer any **four** of the following questions: 5×4=20

a) In a class of 80 students, 50 students know English, 55 know French and 46 know German language. 37 students know English and French, 28 students know French and German, 7 students know none of the languages. How many know only one language?

b) i) For any relation R on a set A, prove that $R\cup R^{-1}$ is a symmetric relation.

ii) If R = {(3, 3), (2, 4), (1, 2)} and S = {(4, 2), (2, 4), (1, 3)}, then find the sets $R\cap S$, $R-S$, $S-R$. 2+3=5

c) Let S be the set of all lines in 3-space. A relation ρ is defined on S by "$l \rho m$ if and only if l lies on the plane of m" for $l, m \in S$. Examine if ρ is (i) reflexive (ii) symmetric (iii) transitive.

d) Find an equivalent expression for $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$ that contains minimal complete set of connectives.

e) If A, B, C are subsets of a universal set S, and if $(A \cup B) - (A \cap B) = (A \cup C) - (A \cap C)$, then prove that $B = C$.

f) i) Find the truth value of $\forall x$, $P(x)$ where $P(x)$ is the statement "$3x+1<10$" and the domain is the set $\{0, 1, 2, 3\}$.

ii) Using truth table show that:

$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$. 2+3=5

UNIT–III

3. Answer any **one** of the following questions: $10 \times 1 = 10$

a) i) Use contrapositive to rewrite the following statement in "if then" form: "Ram will stand first in the class only if he works 12 hours a day".

ii) Construct a truth table for the statement form: $(p \wedge q) \vee \sim r$.

iii) Determine the truth value of the following quantified predicates $\forall x p(x)$, where $p(x)$: "$|x| = x$" on the set of real numbers $\mathbb{R}$. 3+5+2=10

b) i) Write down the converse, inverse and contrapositive of the following statements:

If ABC is a right-angle triangle then $|AB|^2 + |BC|^2 = |AC|^2$.

ii) If $n(U) = 100$, $n(A) = 60$, $n(B) = 50$, $n(A \cap B) = 20$ then find $n\{(A-B) \cup (B-A)\}$ where U is universal set.

iii) Consider the following relation R on the set of all square matrices of order 3.

$R = \{(A, B) : A = P^{-1}BP \text{ for some invertible matrices } P\}$.

Prove that R is equivalence relation. 3+3+4=10

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